Recycle Effects on Heat and Mass Transfer Through a Parallel-Plate Channel

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The problem of heat and mass transfer for fully developed flow in a channel with negligible axial conduction or diffusion is known as the Graetz problem. Recently, many investigators have extended the classical Graetz problem with boundary conditions of the first kind to a variety of boundary conditions in different geometries with or without considering the effects of viscous dissipation, chemical reaction, and axial conduction or diffusion (Davis, 1973; Shah and London, 1978; Dang and Steinberg, 1980; Papoutaskis et al., 1980). Beyond the initial concern for single-stream problems, it is natural to speculate on the applicability of these results to multistream or multiphase problems. Since the governing equations in these problems are coupled through the conjugating conditions at the boundaries, we also call them conjugated Graetz problems. Many applications in industry belong to this category of problems. Some examples are heat transfer in heat exchangers or polymer molding, mass transfer in separation processes with or without chemical reaction, and transport in a variety of metallurgical, biomedical, and biochemical systems (Nunge and Gill, 1965; Murkerjee and Davis, 1972; Kim and Cooney, 1976; Davis and Venkatesh, 1979; Papoutsakis and Ramkrishna, 1981a,b).

Yet, none of the problems mentioned above have considered the effects of recycle of the fluid at the ends. Many separation processes and reactors design have been developed in countercurrent operation with internal or external refluxes at both ends. Typical examples are distillation or extraction in rectifying columns, the draft-tube crystallizer, the continuous membrane column, the thermal diffusion column, the mass diffusion column, and the countercurrent centrifuge. Other cases include loop reactors (Marquart and Blenke, 1980; Marquart, 1981), air-lift reactors, and draft-tube bubble columns (Dussap and Gros, 1982; Jones, 1985; Siegel et al., 1986; Miyahara et al., 1986), which are widely used in absorption, fermentation, and polymerization. In these processes, the reflux indeed has much influence on the heat and mass transfer, which in turn plays a significant role on the design, calculation, and operation of the equipment. Recently, some simplifying models have been proposed to study the effects of recycle on the separation efficiency of continuous-contact, countercurrent homogeneous separation processes (Tsai and Yeh, 1985; Yeh et al., 1986a,b).

There are two purposes in this work: first, to study theoretically and experimentally the effect of recycle on the heat and mass transfer; second, to see if recycle is an effective means to augment heat and mass transfer. For simplicity, only heat transfer in a parallel conduit at steady state with a homogeneous fluid in laminar flow is considered. The model developed in this work may easily be extended to turbulent flow in different geometries or to systems with a quasi-homogeneous fluid in which the dispersion is sufficiently fine and is uniformly distributed in the continuous phase. The results show that recycle at the ends has positive effects on the heat and mass transfer for large Graetz number. Moreover, it is found that the lower the ratio of the thicknesses between the forward flow channel and the conduit, the greater the influence on the transfer rate.

Theoretical Formulation

Consider the heat transfer in a parallel conduit with thickness W, length L, and infinite width as shown in Figure 1. An impermeable plate with negligible thickness and thermal resistance is inserted between the walls to divide the conduit into two channels with thickness ΔW and $(1 - \Delta)W$, respectively. The fluid with volume flow rate V and temperature T_o flows steadily

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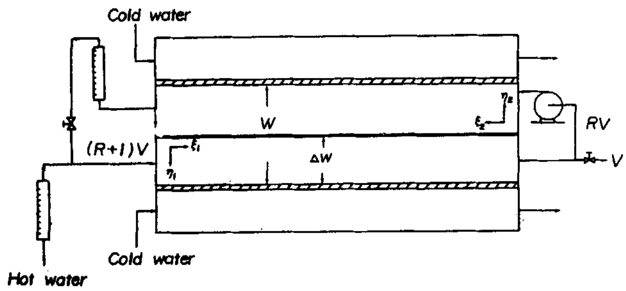

Figure 1. Parallel conduit with refluxes at both ends.

through the conduit. The volume flow rate of recycle RV is reached by means of a conventional pump situated at the outlet. Accordingly, the fluid will mix at the inlet in each channel. Although Figure 1 has external recycle, internal recycle at both ends (Siegel et al., 1986) can be considered.

The analysis is based on the following assumptions:

1. Physical properties and wall temperatures are assumed fixed
2. Purely fully-developed laminar flow exists in each channel
3. End effects and axial diffusion are negligible

With this set of assumptions, the velocity distributions and equation of energy in dimensionless form may be expressed as

$$v_1(\eta) = 6(1+R)\left(\frac{\eta}{\Delta^2} - \frac{\eta^2}{\Delta^3}\right), \quad 0 \le \eta \le \Delta \tag{1}$$

$$v_2(\eta) = \frac{6R}{(1-\Delta)^3}[\eta^2 - (1+\Delta)\eta + \Delta], \quad \Delta \le \eta \le 1 \tag{2}$$

$$v_i(\eta)\frac{\partial\theta_i(\eta,\xi)}{\partial\xi} = \frac{\partial\theta_i^2(\eta,\xi)}{\partial\eta^2}, \quad i = 1, 2 \tag{3}$$

in which

$$\eta = \frac{y}{W}, \quad \xi = \frac{\alpha x}{U_b W^2} = \frac{x}{LG_z}, \quad \theta_i = \frac{T_i - T_o}{T_s - T_o}, \quad U_b = \frac{V}{BW}, \quad v_i = \frac{U_i}{U_b} \tag{4}$$

The boundary conditions for solving Eq. 3 are

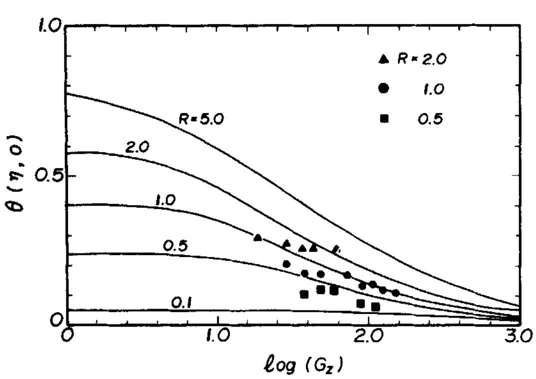

Figure 2. Theoretical and experimental dimensionless inlet temperature of fluid after mixing.
Reflux ratio as parameter; $\Delta = 0.5$, $0 \le \eta \le \Delta$

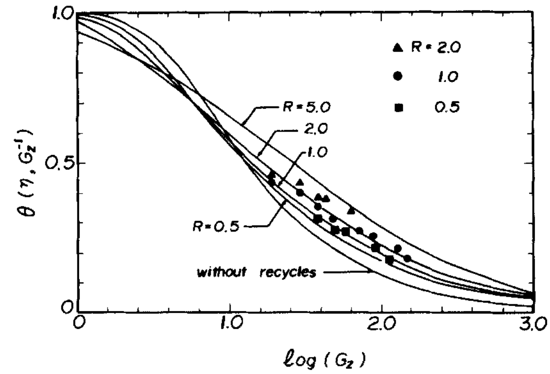

Figure 3. Theoretical and experimental dimensionless average outlet temperature.
Reflux ratio as parameter; $\Delta = 0.5$, $\Delta \le \eta \le 1$

$$\theta_1(0,\xi) = 1 \tag{5}$$

$$\theta_1(\Delta,\xi) = \theta_2(\Delta,\xi) \tag{6}$$

$$\frac{\partial\theta_1(\Delta,\xi)}{\partial\eta} = \frac{\partial\theta_2(\Delta,\xi)}{\partial\eta} \tag{7}$$

$$\theta_2(1,\xi) = 1 \tag{8}$$

$$\theta_1(\eta,0) = \frac{-\int_\Delta^1 v_2(\eta)\,\theta_2(\eta,0)d\eta}{R+1}, \quad 0 \le \eta \le \tag{9}$$

$$\theta_2(\eta, G_z^{-1}) = \frac{\int_0^\Delta v_1(\eta)\,\theta_1(\eta, G_z^{-1})d\eta}{R+1} \quad \Delta \le \eta \le 1 \tag{10}$$

Inspection of the above equations shows that the inlet conditions for both channels are not specified *a priori* and reverse flow occurs. Since there is still no analytical solution to this kind of problem, we will use a numerical method to find the approximate solution.

Numerical Solution

A finite-difference technique was used to solve the above equations. In order to reduce the computation time, the coordinate is changed as shown in Figure 1. The Crank-Nicolson

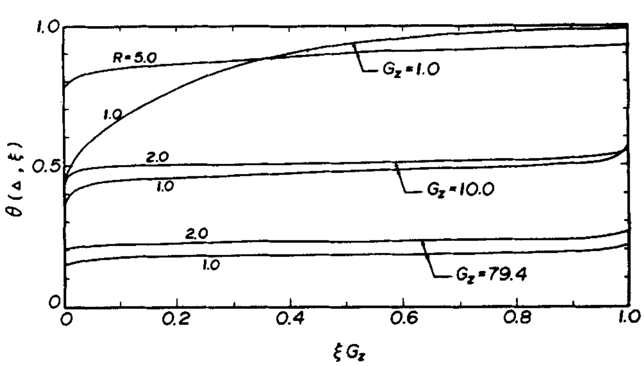

Figure 4. Dimensionless interfacial temperature.
Reflux ratio and Graetz number as parameters; $\Delta = 0.5$

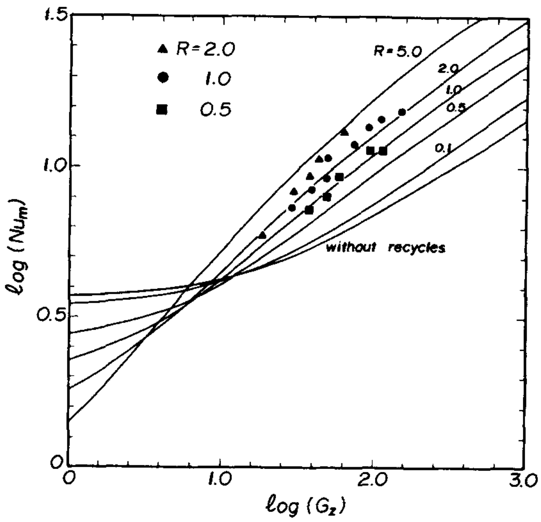

Figure 5. Theoretical and experimental logarithmic average Nusselt number.
Reflux ratio as parameter; $\Delta = 0.5$

implicit formulation was applied to Eq. 3. The algorithm to solve the resultant algebraical equations was much the same as that used by Nunge and Gill (1965), that is:

1. Assume the interfacial temperature distributions to satisfy Eq. 6. In general, there is no rule to make this assumption.

2. Solve the resultant tridiagonal matrixes for both channels and check if the temperature distributions satisfy Eqs. 7, 9, and 10. The derivatives in Eq. 7 are replaced by the five-point forward difference formula used by Nunge and Gill. The relative error is used as the tolerance to test the convergence.

3. Whenever the tolerance for each interfacial point is not satisfied, try a new profile of interfacial temperature. This procedure is repeated until every tolerance is within $\pm 0.1\%$.

Once the temperature distributions are solved, one may calculate the logarithmic average Nusselt number from Eq. 11.

$$Nu_m = -0.5 G_z \ln [1 - \theta_2(\eta, G_z^{-1})] \quad (11)$$

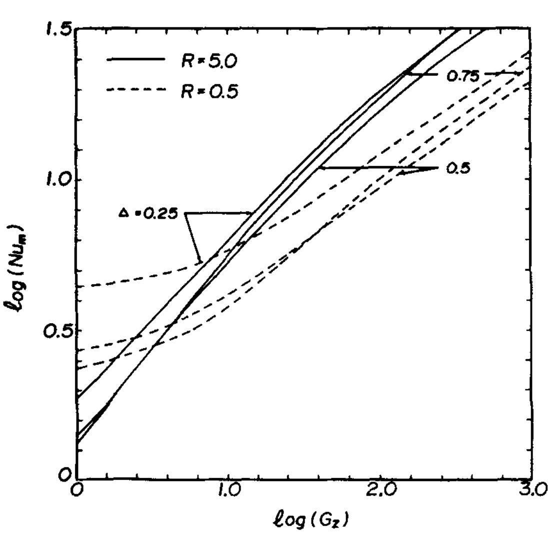

Figure 6. Logarithmic average Nusselt number.
Reflux ratio and Δ as parameters

Some results are shown in Figures 2–6. The dimensionless outlet temperature and logarithmic average Nusselt number for the fluid in the same conduit without the introduction of an impermeable plate and recycles have also been calculated and are represented in Figures 3 and 5 for comparison (Shah and London, 1978).

Experimental Procedure

A parallel conduit of stainless steel with working dimensions 120×4.3 cm was constructed as shown in Figure 1. A stainless steel plate with dimensions $120 \times 20 \times 0.3$ cm is welded at the position with $\Delta = 0.5$. Cooling water was circulated at a high flow rate in the jackets to keep the wall temperature at the mean temperature of 16 ± 0.5°C. Two control valves and a centrifugal pump of 0.25 hp were used to regulate the reflux ratio R, which can be calculated from the rotameters. The equipment was wrapped with foamed plastics. A layer of fiberglass was inserted between the plastic and stainless steel to ensure insulation from the surroundings. Thirteen K-type thermocouples were used to measure the temperature. Eight were used to monitor wall temperature and the other five were inserted at the positions to measure T_o, $T_1(\eta, 0)$, $T_2(\eta, 0)$, $T_2(\eta, G_z^{-1})$, and the outlet temperature.

Hot water with $T_o = 62$°C was introduced and the measurements from the rotameters and recorder were taken after steady state was reached. The volume flow rate was controlled to ensure all fluid in the conduit to be in laminar flow. It was found that the temperature rise across the pump could be neglected. Some results are represented in Figures 2, 3, and 5 with reflux ratio as parameter. As the Graetz number increases, which is equivalent to large flow rate, the hydrodynamic instability due to mixing at the inlet caused the float to the rotameter to oscillate. This behavior was more evident as the reflux ratio increased. Accordingly, the experimental data for large Graetz number are omitted. Moreover, owing to the large range of the rotameter, the results for small Graetz number are also missing.

Results and Discussion

The theoretical and experimental dimensionless inlet temperatures of the fluid after mixing are represented in Figure 2 with the reflux ratio as parameter for $\Delta = 0.5$. For a fixed reflux ratio, decreasing the Graetz number will increase the residence time of the fluid in the column and hence the outlet dimensionless temperature. Therefore, it is concluded that the preheating (or mixing) effect of the inlet fluid increases when the reflux ratio rises or the Graetz number decreases.

Figure 3 shows the theoretical and experimental dimensionless average outlet temperature with the reflux ratio as parameter for $\Delta = 0.5$. It is shown that for a fixed reflux ratio, this temperature decreases with increasing the Graetz number owing to the short residence time of the fluid. Moreover, for a fixed Graetz number the increase of the reflux ratio will also decrease the residence time of the fluid. However, the introduction of reflux still has positive effects on the outlet temperature (or heat transfer) for large Graetz number (i.e., short column or large flow rate). This is due to the preheating effect having more influence than the residence-time effect here. As the Graetz number decreases, the residence-time effect becomes more and more important. Eventually, the preheating effect for large reflux ratio can no longer compensate for the decrease of residence time, and hence the outlet temperature decreases. This behavior is easily verified from the interfacial temperature distribution in Figure 4 with the reflux ratio and the Graetz number as parameters for $\Delta = 0.5$.

The theoretical and experimental logarithmic average Nusselt numbers with and without recycle have been calculated and are represented in Figure 5 with the reflux ratio as parameter for $\Delta = 0.5$. One may also explain this behavior from the competition of preheating and residence-time effects. Therefore, it concluded that recycle can enhance the heat and mass transfer for the fluid with large Graetz number.

Figure 6 shows the influence of the reflux ratio, the ratio of the thicknesses Δ, and the Graetz number on the logarithmic average Nusselt number. The smaller the ratio Δ is, the greater is the value of Nu_m, especially for small reflux ratio. However, this increase in heat and mass transfer might offset the price of increasing the total pressure drop of the conduit. Therefore, a careful calculation of the economics is necessary before designing a suitable value of Δ.

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Notation

- B = conduit width, cm
- G_z = Graetz number, $U_b W^2/\alpha L$
- L = conduit length, cm
- R = reflux ratio, reverse volumn flow rate divided by input volumn flow rate
- T = temperature of fluid, C
- T_o = inlet temperature of fluid in conduit, C
- T_s = wall temperature, C
- U = velocity distribution, cm/s
- U_b = reference velocity, V/WB, cm/s
- V = input volumn flow rate of conduit, cm^3/s
- v = velocity
- W = thickness of conduit, cm
- x = longitudinal coordinate, cm
- y = transversal coordinate, cm

Greek letters

- α = thermal diffusivity of fluid, cm^2/s
- Δ = ratio of thicknesses between forward flow channel and conduit
- η = transversal coordinate, y/W
- θ = temperature, Eq. 4
- ξ = longitudinal coordinate

Subscript

- 1 = forward flow channel
- 2 = backward flow channel

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